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# CONVERGENCE OF S-METRIC SPACE

**G.S.Sao[*1], S.N.Gupta[2] & G.P.Banaj[3]**

[*1]Dept of Mathematics, Govt. ERR PG Science College, Bilaspur (C.G.)

[2&3]Dept.of Mathematics, K. Govt. Arts & Science College, Raigarh (C.G.)

## ABSTRACT

In this paper, we study fixed point theorems in S-metric spaces focusing on single mapping]. we obtain fixed point in S-metric spaces.

**Keywords:** *S-metric space, fixed point.*

***2010 MSC:*** *54H25, 54E99.*

## I. INTRODUCTION

In 2006, Z. Mustafa and B. I. Sims [6] introduced the concept of G-metric space which is a generalization of metric space, and proved some fixed point theorems in G-metric space. Subsequently, many authors were proved fixed point theorems in G- metric space (see, eg. [3,7,11]). And B. C. Dhage [4] introduced the notion of D-metric space. In 2007, S. Sedghi, N. Shobe and H. Zhou [10] introduced D*- metric space which is a modification of D-metric space of [4] and proved some fixed point theorems in D*- metric space and later on many authors were proved fixed point theorems in D*- metric space (see, e.g. [1,5]). In 2012, S. Sedghi et al. [9] introduced the notion of S-metric space which is a generalization of G-metric space of [4] and D*- metric space of [10] and proved some fixed point theorems on S-metric space. Recently, S. Sedghi, N.V. Dung [8] proved generalized fixed point theorems in S-metric spaces which is a generalization of [9]. In this paper, we proved some fixed point results on complete S-metric spaces. Our results extended and improved the results of [8].

## II. PRELIMINARIES

**Definition 2.1.** Let $X$ be a nonempty set. An S-metric on $X$ is a function $S : X^3 \to [0,\infty)$ that satisfies the following conditions holds for all $x, y, z, a \in X$.

1. $S(x,y, z) = 0 \iff x = y = z$
2. $S(x, y, z) \le S(x, x, a) + S(y, y, a) + S(z, z, a)$

The pair $(X, S)$ is called an S-metric space.

**Definition 2.2.** Let $X$ be a nonempty set. A metric on $X$ is a function $d : X^2 \to [0,\infty)$ if there exists a real number $b \ge 1$ such that the following conditions holds for all $x,y, z \in X$.

(1) $d(x, y) = 0 \iff x = y$
(2) $d(x, y) = d(y, x)$
(3) $d(x, z) \le b[d(x, y) + d(y, z)]$

The pair $(X, d)$ is called a b-metric space.

**Definition 2.3** Let $(X, S)$ be an S-metric space a sequence $\{x_n\} \subset X$ *is Cauchy sequence* if $S(x_n , x_n , x_m) \to 0$ as $m,n\to \infty$.That is, for each $\varepsilon > 0$, there exists $n_0 \in N$ such that for all $m,n \ge n_0$ we have $S(x_n , x_n , x) < \varepsilon$ .

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**Definition 2.4.** Let $(X, S)$ be an S-metric space a sequence $\{x_n\} \subset X$ converges to $x \in X$ if $S(x_n\ x_n\ x) \to 0$ as $n \to \infty$. That is, for each $\varepsilon > 0$, there exists $n_0 \in N$ such that for all $n \geq n_0$ we have $S(x_n , x_n , x) < \varepsilon$ ". We write for $x_n \to x$.

**Definition 2.5.** The S-metric space $(X, S)$ is complete if every Cauchy sequence converges.

**Lemma 2.6.** *Let $f : X \to Y$ be a map from an S-metric space $X$ to an S-metric space $Y$ .Then $f$ is continuous at $x \in X$ if and only if $f(x_n) \to f(x)$ whenever $x_n \to x$.*

**Lemma 2.7.** *Let $(X, S)$ be an S-metric space. If $x_n \to x$ and $y_n \to y$ then $S(x_n, x_n, y_n) \to S(x, x, y)$.*

**Lemma 2.8.** *In an S-metric space, we have $S(x, x, y) = S(y, y, x)$ for all $x, y \in X$.*

## III. MAIN RESULT

**Theorem 3.1.** *Let T be a self-map on a complete S-metric space $(X,S)$ and*
$S(Tx, Ty,z) \leq a_1 S(x, y,z)+a_2 [S(x, Tx,z)+ S(y, Ty,z)]+a_3[S(x,Ty,z)+ S(y, Tx,z)]$
*for all $x, y, z \in X$ . Then T has a fixed point. If $a_1+a_2+a_3 < 1/2$ .*

*Proof: Using the definition of S-metric space ,we have*

$$\begin{aligned}S(Tx, Ty,z) &\leq S(Tx,Tx,a)+S(Ty,Ty,a)+S(z,z,a)\\ &\leq a_1[S(x,x,a)+S(y,y,a)+S(z,z,a)\\ &\quad +a_2[S(x,x,a)+S(Tx,Tx,a)+S(z,z,a)+S(y,y,a)+S(Ty,Ty,a)+S(z,z,a)\\ &\quad +a_3[S(s,s,a) +S(Ty,Ty,a)+S(z,z,a)+S(y,y,a)+S(Tx,Tx,a)+S(z,z,a)]\end{aligned}$$

$$\Rightarrow (1-a_2-a_3)[S(Tx,Tx,a)+S(Ty,Ty,a)]+(1-a_1-2a_2-2a_3)S(z,z,a) \leq (a_1+a_2+a_3)[S(x,x,a)+S(y,y,a)]$$

*Applying the given condition $a_1+a_2+a_3 < 1/2$ then we are getting $Tx=Ty=z$ ,hence T has a fixed point*

***Theorem 3.2**: Let T be a self-map on a complete S-metric space $(X,S)$ and*

$$S(Tx,Tx,Ty) \leq aS(Tx,Tx,y)+bS(Ty,Ty,y)$$

*for all $x, y \in X$ then T has a fixed point and continuous , if $a,b \geq 0$ and $a+2b<1$*

*Proof:* S. Sedghi, N.V. Dung [8] introduce
$C_1$:for all $x,y,z \in R_+$, *if* $y \leq S(Tx,Tx,0)$ *with* $z \leq 2x+y$ *then T has a fixed point*
$C_2$ *:for all* $y \in R_+$ *, if* $y \leq S(Ty,0,Ty)$ *then* $y=0$ *and T has a fixed point and which would be unique*
$C_3$ *: if* $x_i \leq y_i+z_i$ *for all* $x,y,z \in R_+$ *and* $i \leq 3$ *then* $S(Tx_1,Tx_2,Tx_3) \leq S(Ty_1,Ty_2,Ty_3)+S(Tz_1,Tz_2,Tz_3)$

*Then T has a fixed point which would be continuous.*
*Here Suppose* $S(Tx,Ty,Tz)=ax+by$ $a,b>0,a+2b<1$ ; $x,y,z \in R_+$, *then*

$$S(Tx,Ty,0)=ax+b(x+y)$$

*If* $y \leq S(Tx,Tx,0)$ *with* $z \leq 2x+y$

$$\begin{aligned}y &\leq ax+bx+by\\ &\leq (a+b)x +by\end{aligned}$$

*So* $(1-b)y \leq (a+b)x$

$$y \leq \frac{a+b}{1-b}x \quad \text{but } a+2b<1 \quad \text{then} \quad \frac{a+b}{1-b}<1$$

*Therefore S satisfies $C_1$ then T has a fixed point*

*Suppose* $y \le S(Ty,0,y) \le ay+b(0+0)= ay$
*Then* $y=0, a<1$

*Therefore S satisfied* $C_2$ *then T has a fixed point and which would be unique*

*Fnally if* $x_i \le y_i+z_i$ *for* $i \le 3$ *then*
$S(Tx_1,Tx_2,Tx_3)=ax_1+bx_2 = a(y_1+z_1)+b(y_2+z_2) = ay_1+by_2+az_1+bz_2$
$\le S(Ty_1,Ty_2,Ty_3)+S(Tz_1,Tz_2,Tz_3)$

*More over* $S(0,0,0)=0+b(0+2y)=2by$ *where* $2b<1$

*Therefore S satisfies* $C_3$ *then T has a fixed point which would be continuous.*

*Hence T is continuous and T has a fixed point which is unique.*

## IV. CONCLUSION

We have come to conclusion that T has a fixed point which is unique .

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